2015 MFE Programming Workshop Lab 5

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1 Implied Volatility

Last week you created a function to calculate the Black-Scholes value of an option (if you didn't create this function then you can use `blsprice` from the financial toolbox). One of the parameters used by this function is the volatility σ. Often times you will want to use option prices to back out the volatility that is implied by these prices. Given that the other parameters of the model are not particularly difficult to estimate, this becomes a root finding problem. Specifically if the Black-Scholes price of an option is given by $P_{BS}(r, T, K, S_0, \sigma)$ and the observed price is P_{obs}, then the Black-Scholes implied volatility is σ such that $P_{BS} = P_{obs}$.

The file *OptionPrices.xlsx* contains 3 columns: the strike price, observed price, and implied volatility for options on the SPY as of November 14, 2014 as given by Yahoo finance. Keeping all parameters as yearly, use $r = .01$, $S_0 = 204.24$, and $T = 30/252$ since these options have 30 trading days until expiration.

- Read the data in from the excel file and use a root finding algorithm to calculate the Black-Scholes implied volatility for each option.
- Create two plots:
  - One with two panels. The first will plot the strike price versus the Yahoo finance implied volatility. The second panel will have your implied volatility versus the strike price.
  - The second plot will have both implied volatilities versus the strike price.
- If you did things right, you should see a nice smile on the charts. This is know as the volatility smile, and I'm sure this isn't the last time you'll be hearing about it!

2 Mean-Variance Portfolio Optimization

The file *stockdata.csv* contains daily stock prices P_{it} for 9 stocks, where i indexes the stock and t indexes the time period. The returns on these stocks are given by $R_{it} = \frac{P_{it}}{P_{it-1}} - 1$. Given a set of portfolio weights (represented as an N-vector) w, the mean return on this portfolio is $w'\mu$ where μ is the Nx1 vector of sample means of the asset returns. The variance of this portfolio is given by $w'\Sigma w$ where Σ is the sample variance-covariance matrix of the asset returns. Since this is a quadratic programming problem we will use the matlab function `quadprog` to find various portfolio weights.

- Read the prices in from the csv file and calculate the daily returns. Calculate the sample mean vector and the sample covariance matrix of the returns.

- Calculate the weights of the minimum variance portfolio constraining the weights to sum to 1.
- Find the weights of the minimum variance portfolio such that the mean daily return is equal to 0.002.
- Suppose that you are not allowed to short in your portfolio (that is, have negative weights). Find the weights of the minimum variance portfolio such that the mean daily return is equal to 0.002 and such that the portfolio weights are positive.
- Can you find a portfolio that achieves a daily return of 1% without shorting?