Programming Workshop Lab 1

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1 Call Options

A call option on a stock S, maturing at time T, struck at K is the ability, but not requirement, to purchase an asset S at some point T at the price K. When a trader can borrow and lend as they like at rate r and the price of S can be written at time t as

$$S_t = S_0 e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma B_t}$$

where B_t is normal with mean zero and variance t, and μ and σ are some fixed values.

Let $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}x^2} dx$ denote the CDF of the standard normal distribution. The famous Black-Scholes (Black and Scholes (1973), Merton (1973)) formula (which you will be taught how to derive in your derivatives class) says that a price of a call option on S maturing in T struck at K is

$$S_0 \times \Phi(d_1) - e^{-rT} K \times \Phi(d_2)$$

where

$$d1 = \frac{\log(S_0/K) + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$
$$d2 = \frac{\log(S_0/K) + \left(r - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$$

2 Questions

- Write an R function that takes parameters r, T, K, S_0, and σ and computes the Black-Scholes call price. You will somehow need to evaluate $\Phi(x)$. There are a couple of ways to do this, but all should start with considering how to find the appropriate function.
- Evaluate the above function for the values $T = 1$, $r = .04$, $\sigma = .25$, $K = 95$, and $S_0 = 100$.

- It may be that we need to do this for many parameters. Compute what the price of a call maturing in $T = 1$ year should be on a stock with current price $S_0 = 100$ and volatility $\sigma = .2$, when the riskless rate of interest is $r = .05$. Write code to do this for every strike $K \in \{75, 76, 77, \ldots, 124, 125\}$, and print the results to the screen. Now suppose that you want to do this for stocks of different maturities also, and that you need to use these prices to conduct some further analysis. For the same S_0, r, and σ, populate a matrix with the prices of an option for strikes and maturities $(K, T) \in \{75, 76, 77, \ldots, 124, 125\} \times \{1/12, 2/12, \ldots 23/12, 2\}$. Again compare your results with the results from the built-in R function.

References

Black, Fischer, and Myron Scholes, 1973, The pricing of options and corporate liabilities, *The journal of political economy* pp. 637–654.

Merton, Robert C, 1973, Theory of rational option pricing, *The Bell Journal of Economics and Management Science* pp. 141–183.